# 4.2 Conservation of Momentum

**SECTION EXPECTATIONS**

- Define and describe the concepts related to elastic and inelastic collisions and to open and closed energy systems.
- Analyze situations involving the conservation of momentum and apply them quantitatively.
- Investigate the law of conservation of momentum in one and two dimensions.

**KEY TERMS**

- conservation of momentum
- system of particles
- internal force
- external force
- open system
- closed system
- isolated system
- recoil

When the cue ball hits the eight ball in billiards, the eight ball hits the cue ball. When a rock hits the ground, the ground hits the rock. In any collision, two objects exert forces on each other. You can learn more about momentum by analyzing the motion of both objects in a collision.

**Figure 4.4** The game of billiards offers many excellent examples of collisions.

## Newton's Third Law and Momentum

Newton's third law states that "For every action force on object B due to object A, there is a reaction force, equal in magnitude but opposite in direction, acting on object A due to object B." Unlike Newton's second law, which focusses on the motion of one specific object, his third law deals with the interaction between *two* objects. When you apply Newton's third law to collisions, you discover one of the most important laws of physics — the law of conservation of momentum. The following steps, along with the diagram in Figure 4.5, show you how to derive the law of conservation of momentum by applying Newton's third law to a collision between two objects.

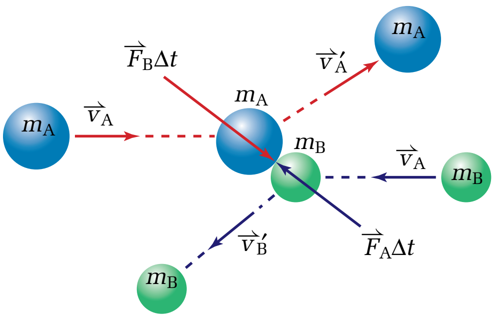



**Figure 4.5** Object A exerts a force on object B, causing a change in B's momentum. At the same time, object B exerts a force equal in size and opposite in direction on object A, changing A's momentum.

- Write the impulse-momentum theorem for each of two objects, A and B, that collide with each other.

$$\vec{F}_A \Delta t = m_A \vec{v}_{A2} - m_A \vec{v}_{A1}$$
$$\vec{F}_B \Delta t = m_B \vec{v}_{B2} - m_B \vec{v}_{B1}$$

- Apply Newton's third law to the forces that A and B exert on each other.

$$\vec{F}_A = -\vec{F}_B$$

- The duration of the collision is the same for both objects. Therefore, you can multiply both sides of the equation above by $\Delta t$.

$$\vec{F}_A \Delta t = -\vec{F}_B \Delta t$$

- Substitute the expressions for change in momentum in the first step into the equation in the third step and then simplify.

$$m_A \vec{v}_{A2} - m_A \vec{v}_{A1} = -(m_B \vec{v}_{B2} - m_B \vec{v}_{B1})$$

- Algebraically rearrange the last equation so that (1) the terms representing the before-collision conditions precede the equals sign and (2) the terms for the after-collision conditions follow the equals sign.

$$m_A \vec{v}_{A2} - m_A \vec{v}_{A1} = -m_B \vec{v}_{B2} + m_B \vec{v}_{B1}$$
$$m_A \vec{v}_{A1} + m_B \vec{v}_{B1} = m_A \vec{v}_{A2} + m_B \vec{v}_{B2}$$

The last equation is a mathematical expression of the law of **conservation of momentum,** which states that the total momentum of two objects before a collision is the same as the total momentum of the same two objects after they collide.

### LAW OF CONSERVATION OF MOMENTUM

The sum of the momenta of two objects before collision is equal to the sum of their momenta after they collide.

$$m_A \vec{v}_A + m_B \vec{v}_B = m_A \vec{v}\,'_A + m_B \vec{v}\,'_B$$

| Quantity | Symbol | SI unit |
|---|---|---|
| mass of object A | $m_A$ | kg (kilograms) |
| mass of object B | $m_B$ | kg (kilograms) |
| velocity of object A before the collision | $\vec{v}_A$ | $\frac{m}{s}$ (metres per second) |
| velocity of object B before the collision | $\vec{v}_B$ | $\frac{m}{s}$ (metres per second) |
| velocity of object A after the collision | $\vec{v}\,'_A$ | $\frac{m}{s}$ (metres per second) |
| velocity of object B after the collision | $\vec{v}\,'_B$ | $\frac{m}{s}$ (metres per second) |

**PHYSICS FILE**

When working with collisions, instead of using subscripts such as "2," physicists often use a superscript symbol called a "prime," which looks like an apostrophe, to represent the variables *after* a collision. The variable is said to be "primed." Look for this notation in the box on the left.

## MISCONCEPTION

**A Closed System Is Not Isolated**

Many people confuse the terms "closed" and "isolated" as they apply to systems. Although it might sound as though closed systems would not exchange anything with their surroundings, they do allow energy to enter or leave. Only isolated systems prevent the exchange of energy with the surroundings.

The law of conservation of momentum can be broadened to more than two objects by defining a **system of particles**. Any group of objects can be defined as a system of particles. Once a system is defined, forces are classified as internal or external forces. An **internal force** is any force exerted on any object in the system due to another object in the system. An **external force** is any force exerted by an object that is not part of the system on an object within the system.

Scientists classify systems according to their interaction with their surroundings, as illustrated in Figure 4.6. An **open system** can exchange both matter and energy with its surroundings. Matter does not enter or leave a **closed system,** but energy can enter or leave. Neither matter nor energy can enter or leave an **isolated system.**

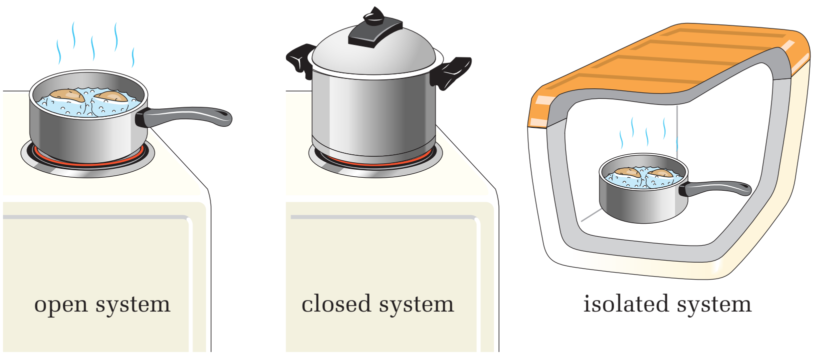



**Figure 4.6** An open pot of potatoes boiling on the stove represents an open system, because heat is entering the pot and water vapour is leaving the system. A pressure cooker prevents any matter from escaping but heat is entering, so the pressure cooker represents a closed system. If the pot is placed inside a perfect insulator, neither heat nor water can enter or leave the system, making it an isolated system.

A force can do work on a closed system, thus increasing the energy of the system. Clearly, if no external forces can act on a system, it is isolated. To demonstrate that the momentum of an isolated system is conserved, start with the impulse-momentum theorem, where $\vec{p}_{sys}$ represents the total momentum of all of the objects within the system.

- An impulse on a system due to an external force causes a change in the momentum of the system. $$\vec{F}_{ext}\Delta t = \Delta\vec{p}_{sys}$$
- If a system is isolated, the net external force acting on the system is zero. $$(0.0\ \text{N})\ \Delta t = |\Delta\vec{p}_{sys}|$$
- If the impulse is zero, the *change* in momentum must be zero. $$|\Delta\vec{p}_{sys}| = 0.0\ \frac{\text{kg}\cdot\text{m}}{\text{s}}$$

**COURSE CHALLENGE**

**Momentum**

A moving planet, bowling ball, or an electron share the property of momentum. Learn more about momentum conservation as it relates to your *Course Challenge* on page 603 of this text.

The last expression is an alternative form of the equation for the conservation of momentum. The equation states that the change in momentum of an isolated system is zero. The particles or objects within the system might interact with each other and exchange momentum, but the total momentum of the isolated system does not change.

In reality, systems are rarely perfectly isolated. In nearly all real situations, immediately after a collision, frictional forces and interactions with other objects change the momentum of the objects involved in the collision. Therefore, it might appear that the law of conservation of momentum is not very useful. However, the law always applies to a system from the instant before to the instant after a collision. If you know the conditions just before a collision, you can always use conservation of momentum to determine the momentum and, thus, velocity of an object at the instant after a collision. Often, these values are all that you need to know.

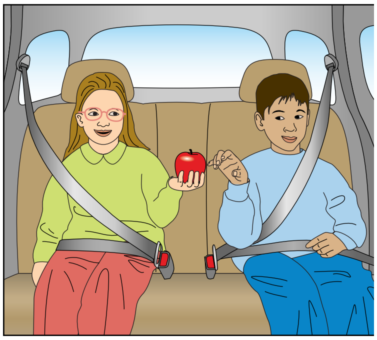


**Figure 4.7** A moving car and its occupants can be defined as being a system. The children in the car might be exerting forces on each other or on objects that they are handling. Although they are exchanging momentum between themselves and the objects, these changes have no effect on the total momentum of the system.

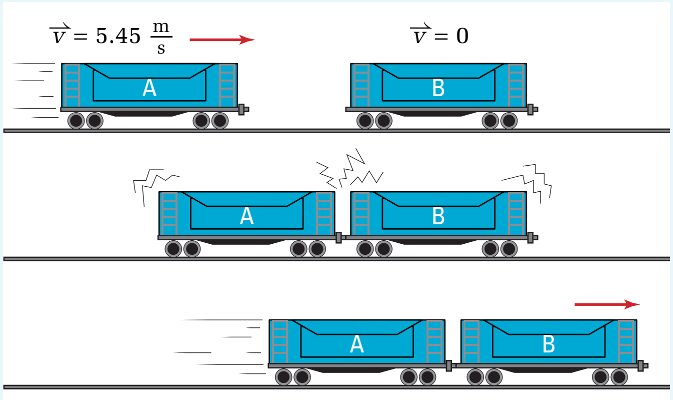
## Collisions in One Dimension

Since momentum is a vector quantity, both the magnitude and the direction of the momentum must be conserved. Therefore, momentum is conserved in each dimension, *independently*. For complex situations, it is often convenient to separate the momentum into its components and work with each dimension separately. Then you can combine the results and find the resultant momentum of the objects in question. Solving problems that involve only one dimension is good practice for tackling more complex problems.

**SAMPLE PROBLEM**

### Analyzing a Collision between Boxcars

**A $1.75 \times 10^4$ kg boxcar is rolling down a track toward a stationary boxcar that has a mass of $2.00 \times 10^4$ kg. Just before the collision, the first boxcar is moving east at 5.45 m/s. When the boxcars collide, they lock together and continue down the track. What is the velocity of the two boxcars immediately after the collision?**

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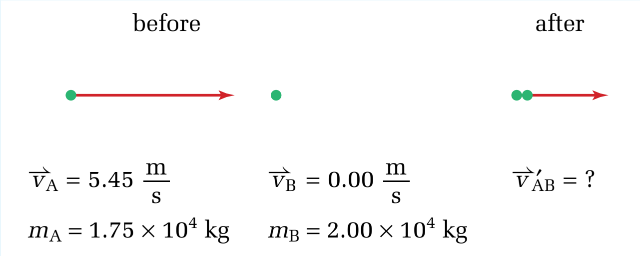
## Conceptualize the Problem

- Make a sketch of the *momentum vectors* representing conditions just *before* and just *after* the collision.
- Before the collision, only *one boxcar* (A) is *moving* and therefore has *momentum*.
- At the instant of the collision, *momentum is conserved*.
- After the collision, the *two boxcars* (A and B) *move as one mass*, with the *same velocity*.




### Identify the Goal

The velocity, $\vec{v}\,'_{AB}$, of the combined boxcars immediately after the collision

## Identify the Variables and Constants

| **Known** | **Implied** | **Unknown** |
|---|---|---|
| $m_A = 1.75 \times 10^4$ kg | $\vec{v}_B = 0.00\ \frac{m}{s}$ | $\vec{v}\,'_{AB}$ |
| $m_B = 2.00 \times 10^4$ kg | | |
| $\vec{v}_A = 5.45\ \frac{m}{s}$[E] | | |

## Develop a Strategy

Apply the law of conservation of momentum.

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B$$

After the collision, the two masses act as one, with one velocity. Rewrite the equation to show this condition.

$$m_A\vec{v}_A + m_B\vec{v}_B = (m_A + m_B)\vec{v}\,'_{AB}$$

Solve for $\vec{v}\,'_{AB}$.

$$\vec{v}\,'_{AB} = \frac{m_A\vec{v}_A + m_B\vec{v}_B}{(m_A + m_B)}$$

Substitute values and solve.

$$\vec{v}\,'_{AB} = \frac{(1.75 \times 10^4\ \text{kg})(5.45\ \frac{m}{s}[E]) + (2.00 \times 10^4\ \text{kg})(0.00\ \frac{m}{s})}{(1.75 \times 10^4\ \text{kg} + 2.00 \times 10^4\ \text{kg})}$$

$$\vec{v}\,'_{AB} = \frac{9.53\ 75 \times 10^4\ \frac{\cancel{kg}\cdot m}{s}[E]}{3.75 \times 10^4\ \cancel{kg}}$$

$$\vec{v}\,'_{AB} = 2.543\ \frac{m}{s}[E]$$

$$\vec{v}\,'_{AB} \cong 2.54\ \frac{m}{s}[E]$$

The locked boxcars were rolling *east* down the track at 2.54 m/s.

## Validate the Solution

The combined mass of the boxcars was nearly double the mass of the boxcar that was moving before the collision. Since the exponents of mass and velocity are always one, making the relationships linear, you would expect that the velocity of the combined boxcars would be just under half of the velocity of the single boxcar before the collision. Half of 5.45 m/s is approximately 2.7 m/s. The calculated value of 2.54 m/s is very close to what you would expect.

## PRACTICE PROBLEMS

8. Claude and Heather are practising pairs skating for a competition. Heather (47 kg) is skating with a velocity of 2.2 m/s. Claude (72 kg) is directly behind her, skating with a velocity of 3.1 m/s. When he reaches her, he holds her waist and they skate together. At the instant after he takes hold of her waist, what is their velocity?

9. Two amusement park "wrecker cars" are heading directly toward each other. The combined mass of car A plus driver is 375 kg and it is moving with a velocity of +1.8 m/s. The combined mass of car B plus driver is 422 kg and it is moving with a velocity of −1.4 m/s. When they collide, they attach and continue moving along the same straight line. What is their velocity immediately after they collide?

## Recoil

Imagine yourself in the situation illustrated in Figure 4.8. You are in a small canoe with a friend and you decide to change places. Assume that the friction between the canoe and the water is negligible. While the canoe is not moving in the water, you very carefully stand up and start to take a step. You suddenly have the sense that the boat is moving under your feet. Why?

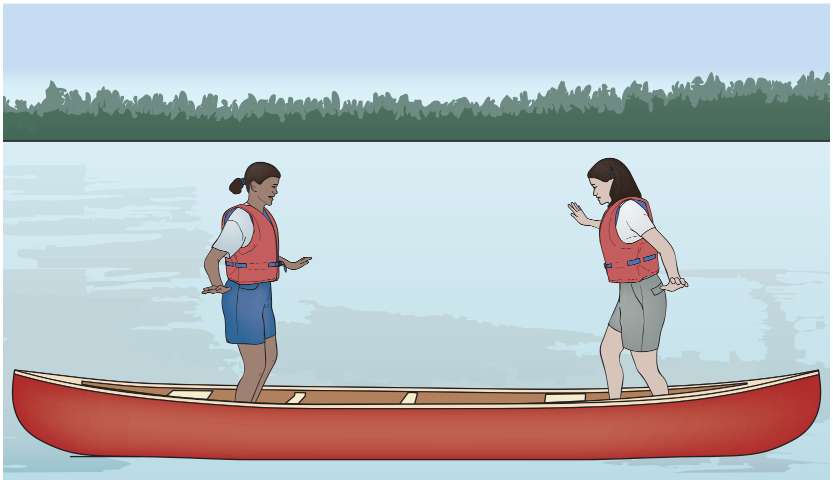


**Figure 4.8** If you start to step forward in a canoe, the canoe recoils under your feet.

When you stepped forward, your foot pushed against the bottom of the canoe and you started to move. You gained momentum due to your velocity. Momentum of the system — you, your friend, and the canoe — must be conserved, so the canoe started to move in the opposite direction. The interaction that occurs when two stationary objects push against each other and then move apart is called **recoil.** You can use the equation for conservation of momentum to solve recoil problems, as the following problem illustrates.

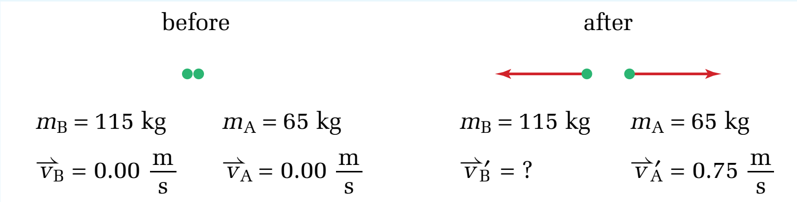
SAMPLE PROBLEM

## Recoil of a Canoe

**For the case described in the text, find the velocity of the canoe and your friend at the instant that you start to take a step, if your velocity is 0.75 m/s[forward]. Assume that your mass is 65 kg and the combined mass of the canoe and your friend is 115 kg.**

### Conceptualize the Problem

- Make a simple sketch of the conditions before and after you took a step.




- The canoe was *not moving* when you started to take a step.
- You gained *momentum* when you started to *move*. Label yourself "A" and consider the direction of your motion to be *positive*.
- The canoe had to *move* in a *negative direction* in order to conserve momentum. Label the canoe and your friend "B."

#### Identify the Goal

The initial velocity, $\vec{v}'_B$ of the canoe and your friend

### Identify the Variables and Constants

| Known | | Implied | Unknown |
|---|---|---|---|
| $m_A = 65$ kg | $\vec{v}'_A = 0.75 \frac{m}{s}$ | $\vec{v}_A = 0.00 \frac{m}{s}$ | $\vec{v}'_B$ |
| $m_B = 115$ kg | | $\vec{v}_B = 0.00 \frac{m}{s}$ | |

### Develop a Strategy

Apply conservation of momentum.

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}'_A + m_B\vec{v}'_B$$

Velocities before the interaction were zero; therefore, the total momentum before the interaction was zero. Set these values equal to zero and solve for the velocity of B after the reaction.

$$0.0 \frac{\text{kg} \cdot \text{m}}{\text{s}} = m_A\vec{v}'_A + m_B\vec{v}'_B$$

$$m_A\vec{v}'_A = -m_B\vec{v}'_B$$

$$\vec{v}'_B = -\frac{m_A\vec{v}'_A}{m_B}$$

Substitute values and solve.

$$\vec{v}'_B = -\frac{(65\ \cancel{\text{kg}})(0.75\ \frac{\text{m}}{\text{s}})}{115\ \cancel{\text{kg}}}$$

$$\vec{v}'_B = -0.4239 \frac{\text{m}}{\text{s}}$$

$$\vec{v}'_B \cong -0.42 \frac{\text{m}}{\text{s}}$$

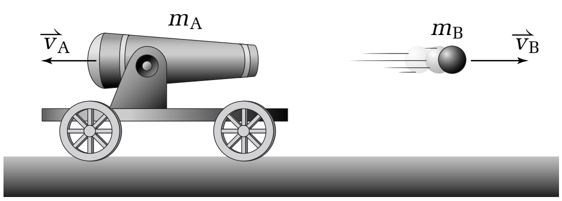
The velocity of the canoe and your friend, immediately after you started moving, was –0.42 m/s.

## Validate the Solution

Since the mass of the canoe plus your friend was larger than your mass, you would expect that the magnitude of their velocity would be smaller, which it was. Also, the direction of the velocity of the canoe plus your friend must be negative, that is, in a direction opposite to your direction. Again, it was.

### PRACTICE PROBLEMS

**10.** A 1385 kg cannon containing a 58.5 kg cannon ball is on wheels. The cannon fires the cannon ball, giving it a velocity of 49.8 m/s north. What is the initial velocity of the cannon the instant after it fires the cannon ball?




**11.** While you are wearing in-line skates, you are standing still and holding a 1.7 kg rock. Assume that your mass is 57 kg. If you throw the rock directly west with a velocity of 3.8 m/s, what will be your recoil velocity?

**12.** The mass of a uranium-238 atom is $3.95 \times 10^{-25}$ kg. A stationary uranium atom emits an alpha particle with a mass of $6.64 \times 10^{-27}$ kg. If the alpha particle has a velocity of $1.42 \times 10^{4}$ m/s, what is the recoil velocity of the uranium atom?

## Collisions in Two Dimensions

Very few collisions are confined to one dimension, as anyone who has played billiards knows. Nevertheless, you can work in one dimension at a time, because momentum is conserved in each dimension independently. For example, consider the car crash illustrated in Figure 4.9. Car A is heading north and car B is heading east when they collide at the intersection. The cars lock together and move off at an angle. You can find the total momentum of the entangled cars because the component of the momentum to the north must be the same as car A's original momentum. The eastward component of the momentum must be the same as car B's original momentum. You can use the Pythagorean theorem to find the resultant momentum, as shown in the following problems.

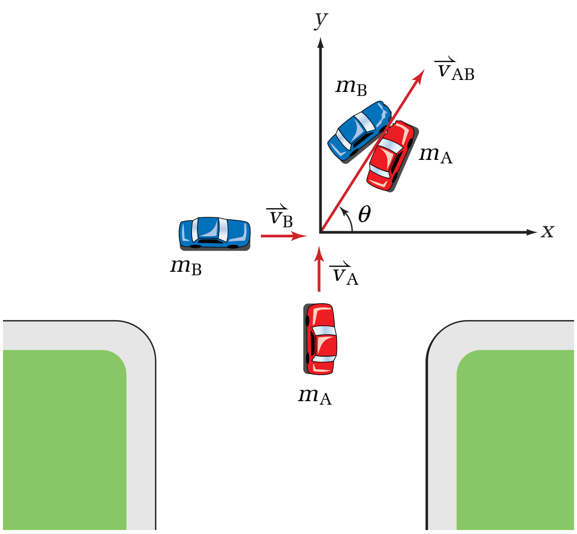
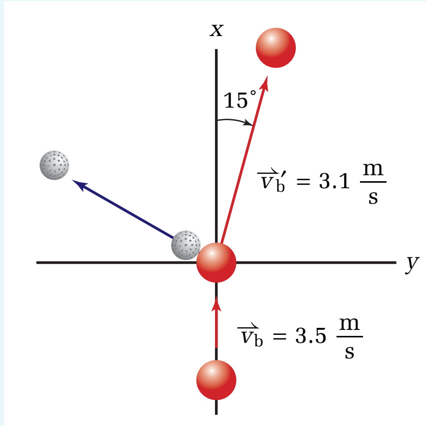
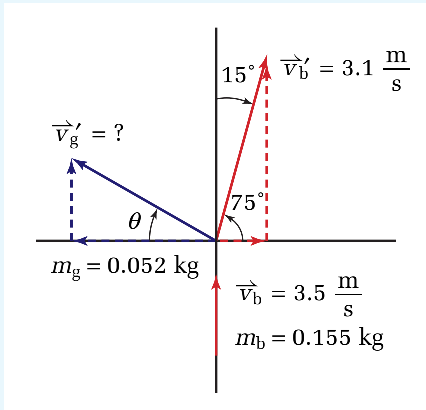



**Figure 4.9** Momentum is conserved independently in both the north-south dimension and the east-west dimension.

## Applying Conservation of Momentum in Two Dimensions

**1. A billiard ball of mass 0.155 kg is rolling directly away from you at 3.5 m/s. It collides with a stationary golf ball of mass 0.052 kg. The billiard ball rolls off at an angle of 15° clockwise from its original direction with a velocity of 3.1 m/s. What is the velocity of the golf ball?**




### Conceptualize the Problem

- Sketch the vectors representing the momentum of the billiard ball and the golf ball immediately before and just after the collision. It is always helpful to superimpose an $x$–$y$-coordinate system on the vectors so that the origin is at the point of the contact of the two balls. For calculations, use the angles that the vectors make with the $x$-axis.
- Momentum is *conserved* in the $x$ and $y$ directions *independently*.
- The *total momentum* of the system (billiard ball and golf ball) *before* the collision is carried by the *billiard ball* and is all in the positive $y$ direction.
- *After* the collision, both balls have *momentum* in both the $y$ direction and the $x$ direction.
- Since the *momentum* in the *x direction* was *zero* before the collision, it must be *zero* after the collision. Therefore, the *x-components of the momentum* of the two balls after the collision must be *equal in magnitude* and *opposite in direction*.
- The *sum* of the $y$-components of the two balls *after* the collision must equal the *momentum* of the billiard ball *before* the collision.
- Use subscript "b" for the billiard ball and subscript "g" for the golf ball.

#### Identify the Goal

The velocity, $\vec{v}'_g$, of the golf ball after the collision

### Identify the Variables and Constants

| Known | | Implied | Unknown |
|---|---|---|---|
| $m_b = 0.155$ kg | $\vec{v}_b = 3.5\ \frac{m}{s}$[forward] | $\vec{v}_g = 0.00\ \frac{m}{s}$ | $\vec{v}'_g$ |
| $m_g = 0.052$ kg | $\vec{v}'_b = 3.1\ \frac{m}{s}$[15° clockwise from original] | | |

**PROBLEM TIP**

When you are working with many bits of data in one problem, it is often helpful to organize the data in a table such as the one shown here.

| Object | | $P_x$ | $P_y$ |
|---|---|---|---|
| before | A | | |
| | B | | |
| | total | | |
| after | A | | |
| | B | | |
| | total | | |

## Develop a Strategy

Write the expression for the conservation of momentum in the $x$ direction.

$$m_b v_{bx} + m_g v_{gx} = m_b v'_{bx} + m_g v'_{gx}$$

Note that the $x$-component of the momentum of both balls was zero before the collision. Then solve for the $x$-component of the velocity of the golf ball after the collision.

$$0.0\ \frac{\text{kg}\cdot\text{m}}{\text{s}} = m_b v'_{bx} + m_g v'_{gx}$$

$$m_g v'_{gx} = -m_b v'_{bx}$$

$$v'_{gx} = -\frac{m_b v'_{bx}}{m_g}$$

Substitute values and solve.

$$v'_{gx} = -\frac{(0.155\ \text{kg})(3.1\ \frac{\text{m}}{\text{s}} \cos 75°)}{0.052\ \text{kg}}$$

$$v'_{gx} = -2.3916\ \frac{\text{m}}{\text{s}}$$

Carry out the same procedure for the $y$-components.

$$m_b v_{by} + m_g v_{gy} = m_b v'_{by} + m_g v'_{gy}$$

$$m_b v_{by} + 0.0\ \frac{\text{kg}\cdot\text{m}}{\text{s}} = m_b v'_{by} + m_g v'_{gy}$$

$$m_g v'_{gy} = m_b v'_{by} - m_b v_{by}$$

$$v'_{gy} = \frac{m_b v_{by} - m_b v'_{by}}{m_g}$$

$$v'_{gy} = \frac{(0.155\ \text{kg})(3.5\ \frac{\text{m}}{\text{s}}) - (0.155\ \text{kg})(3.1\ \frac{\text{m}}{\text{s}} \sin 75°)}{0.052\ \text{kg}}$$

$$v'_{gy} = 1.507\ \frac{\text{m}}{\text{s}}$$

Use the Pythagorean theorem to find the magnitude of the resultant velocity vector of the golf ball.

$$|\vec{v}'_g|^2 = v'^2_{gx} + v'^2_{gy}$$

$$|\vec{v}'_g|^2 = \left(-2.3916\ \frac{\text{m}}{\text{s}}\right)^2 + \left(1.507\ \frac{\text{m}}{\text{s}}\right)^2$$

$$|\vec{v}'_g|^2 = 5.7198\ \frac{\text{m}^2}{\text{s}^2} + 2.271\ \frac{\text{m}^2}{\text{s}^2}$$

$$|\vec{v}'_g|^2 = 7.9908\ \frac{\text{m}^2}{\text{s}^2}$$

$$|\vec{v}'_g| = 2.8268\ \frac{\text{m}}{\text{s}}$$

$$|\vec{v}'_g| \cong 2.8\ \frac{\text{m}}{\text{s}}$$

Use the tangent function to find the direction of the velocity vector.

$$\tan\theta = \frac{v_{gy}}{v_{gx}}$$

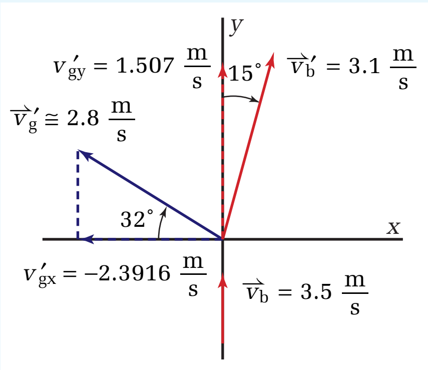
Since the $x$-component is negative and the $y$-component is positive, the vector is in the second quadrant. Use positive values to find the magnitude of the angle from the $x$-axis.

$$\tan\theta = \frac{1.507\ \frac{\text{m}}{\text{s}}}{2.3916\ \frac{\text{m}}{\text{s}}}$$

$$\theta = \tan^{-1} 0.6301$$

$$\theta = 32.22°$$

$$\theta \cong 32°$$




Since the $x$-component is negative and the $y$-component is positive, the resultant vector lies in the second quadrant and the angle is measured clockwise from the $x$-axis.

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The velocity of the golf ball after the collision is 2.8 m/s at 32° clockwise from the negative $x$-axis. (At more advanced levels, you will be expected to report angles counterclockwise from the positive $x$-axis. In this case, the angle would be $180° - 32° = 148°$ counterclockwise from the $x$-axis.)

## Validate the Solution

Since all of the momentum before the collision was in the positive $y$ direction, the $y$-component of momentum after the collision had to be in the positive $y$ direction, which it was. Since there was no momentum in the $x$ direction before the collision, the $x$-components of the momentum after the collision had to be in opposite directions, which they were.

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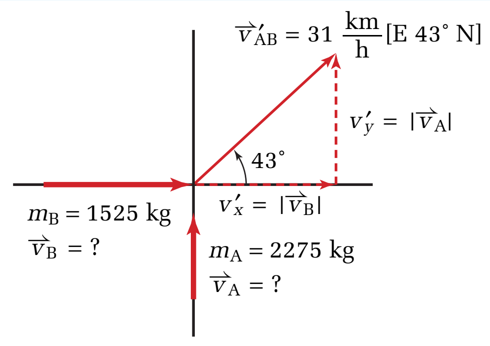
**2. The police are investigating an accident similar to the one pictured in Figure 4.9. Using data tables, they have determined that the mass of car A is 2275 kg and the mass of car B is 1525 kg. From the skid marks and data for the friction between tires and concrete, the police determined that the cars, when they were locked together, had a velocity of 31 km/h at an angle of 43° north of the eastbound street. If the speed limit was 35 km/h on both streets, should one or both cars be ticketed for speeding? Which car had the right of way at the intersection? Was one driver or were both drivers at fault for the accident?**

## Conceptualize the Problem




- Sketch a vector diagram of the momentum before and after the collision.
- Consider the two cars to be a "system." Before the collision, the *north component* of the *momentum* of the system was carried by car A and the *east component* was carried by car B.
- Momentum is conserved in the north-south direction and in the east-west direction independently.
- After the collision, the cars form *one mass* with all of the *momentum*.

### Identify the Goal

The velocities, $\vec{v}_A$ and $\vec{v}_B$, of the two cars before the collision (in order to determine who should be ticketed)

## Identify the Variables and Constants

| Known | | Unknown |
|---|---|---|
| $m_A = 2275$ kg | $\vec{v}'_{AB} = 31\,\frac{km}{h}$[E43°N] | $\vec{v}_A$ |
| $m_B = 1525$ kg | | $\vec{v}_B$ |

**PROBLEM TIP**

In this problem, you have two unknown values, the velocity of car A and the velocity of car B before the collision. To find two unknown values, you need at least two equations. Since momentum is a vector quantity, conservation of momentum provides three equations, one for each dimension. Remember, use as many dimensions as you have unknowns and you will be able to solve momentum problems with as many as three unknowns.

## Develop a Strategy

Write the equation for conservation of momentum.

$$m_A\vec{v}_A + m_B\vec{v}_B = m_A\vec{v}\,'_A + m_B\vec{v}\,'_B$$

Work with the north-south direction only. Modify the equation to show that car B was moving directly east before the crash; its north-south momentum was zero. After the crash, the cars were combined.

$$m_A v_A[N] = (m_A + m_B)v'_{A/B}[N]$$

(Note that vector notations are not included, because you are considering only the north-south component of the velocities.)

Solve the equation for the original velocity of car A.

$$v_A[N] = \frac{(m_A + m_B)v'_{AB}[N]}{m_A}$$

Substitute the values and solve.

$$v_A[N] = \frac{(2275\text{ kg} + 1525\text{ kg})(31\ \frac{\text{km}}{\text{h}}\sin 43^\circ)}{2275\text{ kg}}$$

$$v_A[N] = \frac{(3800\text{ kg})(31\ \frac{\text{km}}{\text{h}})(0.681\ 998\ 4)}{2275\text{ kg}}$$

$$v_A[N] = 35.3\ \frac{\text{km}}{\text{h}}$$

$$\vec{v}_A \cong 35\ \frac{\text{km}}{\text{h}}[N]$$

(Note that the north component of car A's velocity before the crash was the total velocity.)

Carry out the same procedure for the east-west direction of the momentum.

$$m_B v_B[E] = (m_A + m_B)v'_{AB}[E]$$

$$v_B[E] = \frac{(m_A + m_B)v'_{AB}[E]}{m_B}$$

$$v_B[E] = \frac{(2275\text{ kg} + 1525\text{ kg})(31\ \frac{\text{km}}{\text{h}}\cos 43^\circ)}{1525\text{ kg}}$$

$$v_B[E] = 56.49\ \frac{\text{km}}{\text{h}}$$

$$\vec{v}_B \cong 56\ \frac{\text{km}}{\text{h}}[E]$$

Car A was travelling 35 km/h north and car B was travelling 56 km/h east at the instant before the crash. Therefore, car B was speeding and the driver should be ticketed. As well, the driver on the right has the right of way, giving car A the right of way at the intersection. The driver of car B was at fault for the collision. Nevertheless, the driver of car A would have benefited if he or she could have prevented the crash.

## Validate the Solution

The angle at which the locked cars moved after the crash was very close to 45°, which means that the momentum of the two cars before the crash was nearly the same. Car B had a smaller mass than car A, so car B must have moving at a greater speed (magnitude of the velocity), which agrees with the results. Also, the units all cancelled to give km/h, which is correct for velocity.

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## PRACTICE PROBLEMS

**13.** A 0.150 kg billiard ball (A) is rolling toward a stationary billiard ball (B) at 10.0 m/s. After the collision, ball A rolls off at 7.7 m/s at an angle of 40.0˚ clockwise from its original direction. What is the speed and direction of ball B after the collision?

**14.** A bowling ball with a mass of 6.00 kg rolls with a velocity of 1.20 m/s toward a single standing bowling pin that has a mass of 0.220 kg. When the ball strikes the bowling pin, the pin flies off at an angle of 70.0˚ counterclockwise from the original direction of the ball, with a velocity of 3.60 m/s. What was the velocity of the bowling ball after it hit the pin?

**15.** Car A (1750 kg) is travelling due south and car B (1450 kg) is travelling due east. They reach the same intersection at the same time and collide. The cars lock together and move off at 35.8 km/h[E31.6˚S]. What was the velocity of each car before they collided?

## Angular Momentum

Why is a bicycle easy to balance when you are riding, but falls over when you come to a stop? Why does a toy gyroscope, like the one in Figure 4.10, balance on a pointed pedestal when it is spinning, but falls off the pedestal when it stops spinning? The answer lies in the conservation of angular momentum.

**Figure 4.10** When a spinning object begins to fall, its angular momentum resists the direction of the fall.

When an object is moving on a curved path or rotating, it has angular momentum. Angular momentum and linear (or translational) momentum are similar in that they are both dependent on an object's mass and velocity. Analyze Figure 4.11 to find the third quantity that affects angular momentum.

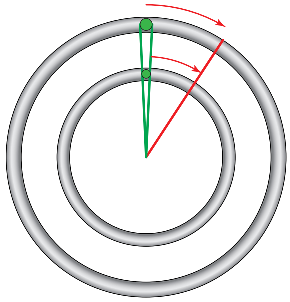

**Figure 4.11** As the distance from the centre of rotation increases, a unit of mass must move faster in order to maintain a constant rate of rotation.

### WEB LINK

***www.mcgrawhill.ca/links/physics12***

For information on current accident-investigation research topics and technological developments related to vehicle safety, go to the above Internet site and click on **Web Links**.